Draft for S-Roux

E. Andó E. Roubin

Last update : 16th February 2018

1 Gauss-Newton Algorithm

1.1 Image transformation

$\boldsymbol{F}$ is an operator that corresponds to the transformation. It reads:

$$\boldsymbol{F} = \begin{bmatrix} 1+a_{xx} & a_{xy} & a_{xz} & t_x \\ a_{yx} & 1+a_{yy} & a_{yz} & t_y \\ a_{zx} & a_{zy} & 1+a_{zz} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

with 12 unknowns. It is an operator on homogeneous coordinates of the image:

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{2}$$

At iteration $n+1$ the increment is noted:

$$\boldsymbol{\delta F}^{(n+1)} = \begin{bmatrix} \delta a_{xx} & \delta a_{xy} & \delta a_{xz} & \delta t_x \\ \delta a_{yx} & \delta a_{yy} & \delta a_{yz} & \delta t_y \\ \delta a_{zx} & \delta a_{zy} & \delta a_{zz} & \delta t_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

so the transformation of the correction is:

$$\boldsymbol{G}^{(n+1)} = \boldsymbol{1} + \boldsymbol{\delta F}^{(n+1)} \tag{4}$$

and since the operator correpsonding top two consecutive transformations $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ is the product $\boldsymbol{F}_2 \cdot \boldsymbol{F}_1 = \boldsymbol{F}_1 \cdot \boldsymbol{F}_2$, we have

$$\boldsymbol{F}^{(n+1)} = \boldsymbol{G}^{(n+1)} \cdot \boldsymbol{F}^{(n)} \tag{5}$$

We note $\tilde{g}^{(n)}(\boldsymbol{x})$ the "corrected" image through the transformation $\boldsymbol{F}$ at iteration n:

$$\tilde{g}^{(n)}(\boldsymbol{x}) \equiv g(\boldsymbol{F}^{(n)} \cdot \boldsymbol{x}) \tag{6}$$

so the transformed image at iteration $n+1$ is written:

$$g(\boldsymbol{F}^{(n+1)} \cdot \boldsymbol{x}) = g(\boldsymbol{G}^{(n+1)} \cdot \boldsymbol{F}^{(n)} \cdot \boldsymbol{x}) = \tilde{g}^{(n)}(\boldsymbol{G}^{(n+1)} \cdot \boldsymbol{x}) \tag{7}$$

1.2 Taylor expension of corrected image

For small h we have:

$$f(x+h) = \sum_{i=0}^{\infty} \frac{x^i}{i!} f^{(i)}(x) = f(x) + xf'(x) + \frac{x^2}{2} f''(x) + \mathcal{O}(x) \tag{8}$$

Apply to $\tilde{g}$ we have the correctied image at iteration $n+1$ that can be approximated by:

$$\tilde{g}^{(n)}(\boldsymbol{G}^{(n+1)} \cdot \boldsymbol{x}) = \tilde{g}^{(n)}(\boldsymbol{x} + \boldsymbol{\delta F}^{(n+1)} \cdot \boldsymbol{x}) \approx \tilde{g}^{(n)}(\boldsymbol{x}) + \boldsymbol{\delta F}^{(n+1)} \cdot \boldsymbol{x} \cdot \boldsymbol{\nabla} \tilde{g}^{(n)}(\boldsymbol{x}) \tag{9}$$